Coordinates and Transformations

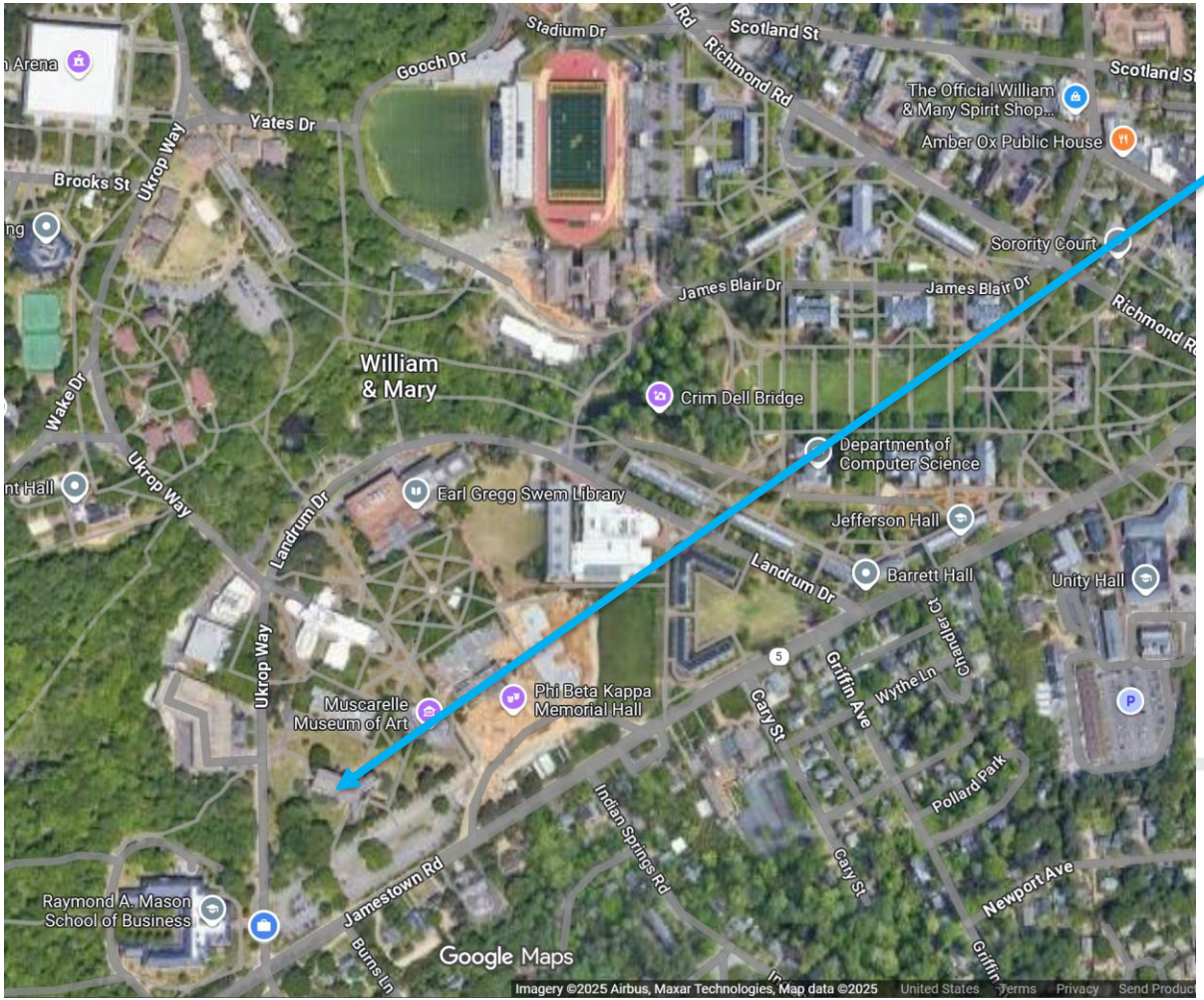
CSCI 420-04 Robotics

WILLIAM & MARY

CHARTERED 1693

Where are you?


Stadium Dr
Scotland St
Gooch Dr
Richmond Rd
The Official William & Mary Spirit Shop...
Amber Ox Public House
Yates Dr
Ukrop Way
Brooks St
Sorority Court
James Blair Dr
James Blair Dr
Richmond Rd
William & Mary
Crim Dell Bridge
Wake Dr
Department of Computer Science
Ukrop Way
Earl Gregg Swem Library
Landrum Dr
Jefferson Hall
Barrett Hall
Unity Hall
Landrum Dr
Chandler Ct
Griffin Ave
Wythe Ln
Ukrop Way
Muscarelle Museum of Art
Phi Beta Kappa Memorial Hall
Cary St
Pollard Park
Indian Springs Rd
Cary St
Newport Ave
Jamestown Rd
Raymond A. Mason School of Business
Burns Ln
Google Maps
Imagery ©2025 Airbus, Maxar Technologies, Map data ©2025
United States
Terms
Privacy
Send Product



37.267390149039755N, -76.71699287853293W

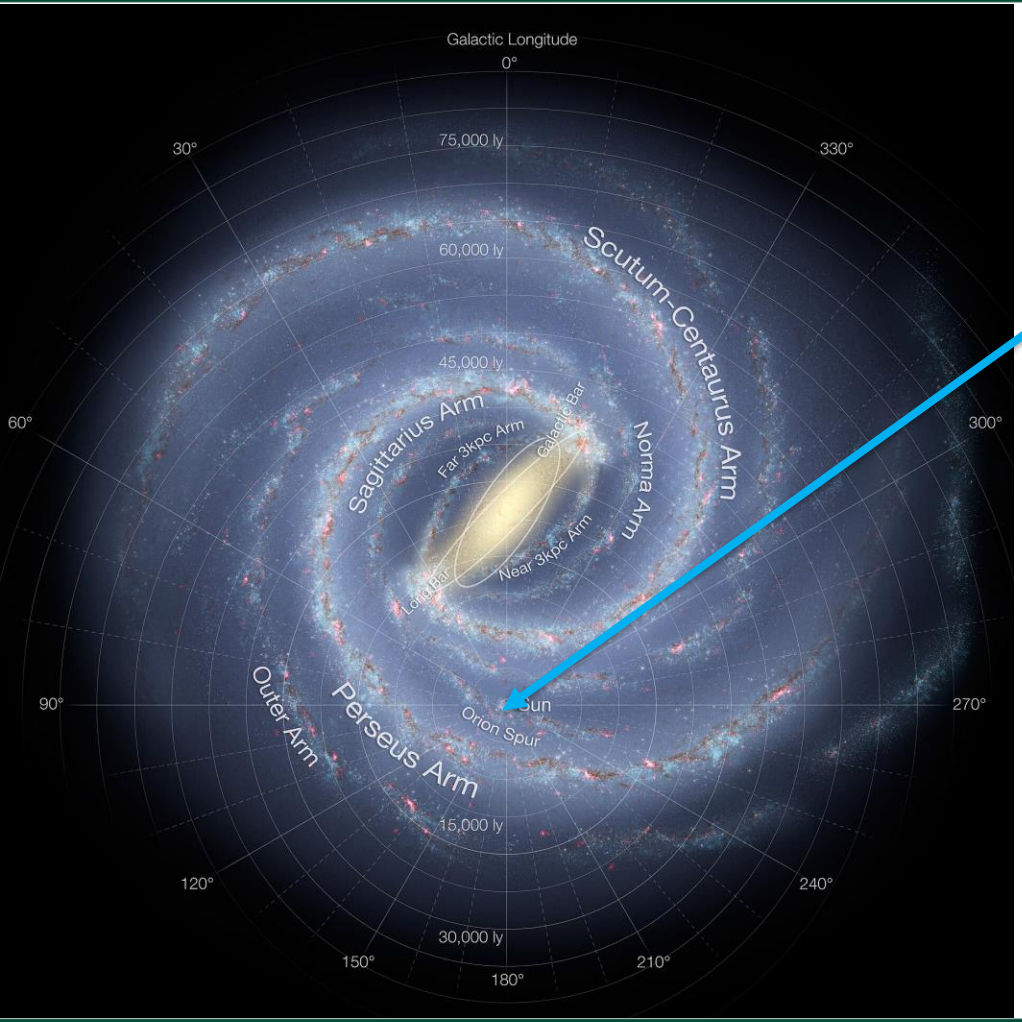
37°16'02.6"N 76°43'01.2"W

778M+X66
Williamsburg, Virginia

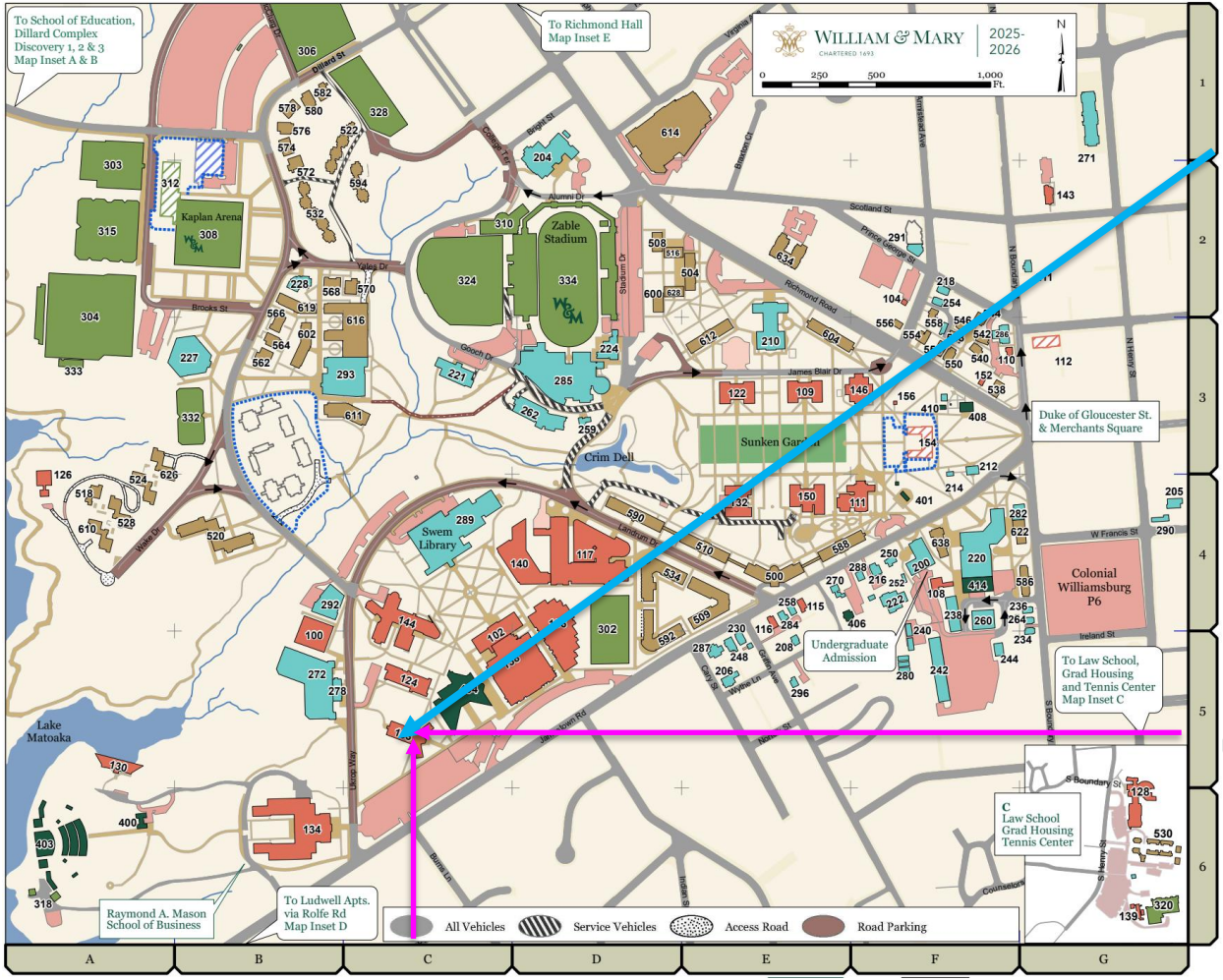
100 Ukrop Way
Williamsburg, Virginia

Boswell Hall Room 341
Williamsburg, Virginia

What else can we use to refer to this place?



0°, 0.00001569ly


To School of Education, Dillard Complex Discovery 1, 2 & 3 Map Inset A & B
To Richmond Hall Map Inset E
WILLIAM & MARY
CHARTERED 1693
2025-2026
0 250 500 1,000 Ft.
N
Kaplan Arena
Zable Stadium
Crim Dell
Swem Library
Sunken Garden
Duke of Gloucester St. & Merchants Square
Colonial Williamsburg P6
Undergraduate Admission
To Law School, Grad Housing and Tennis Center Map Inset C
Lake Matoaka
Raymond A. Mason School of Business
To Ludwell Apts. via Rolfe Rd Map Inset D
C
Law School
Grad Housing
Tennis Center
All Vehicles
Service Vehicles
Access Road
Road Parking
A
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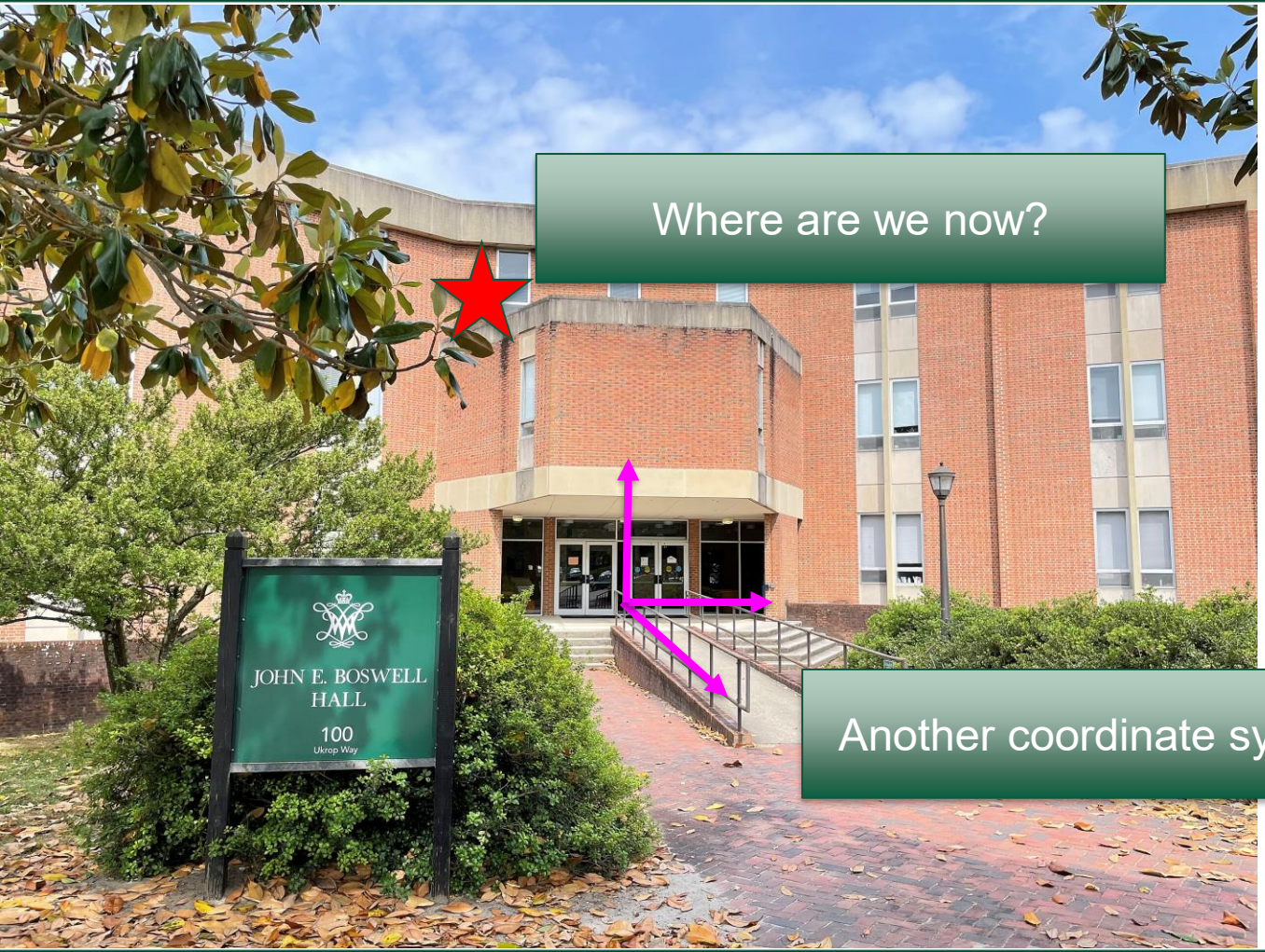
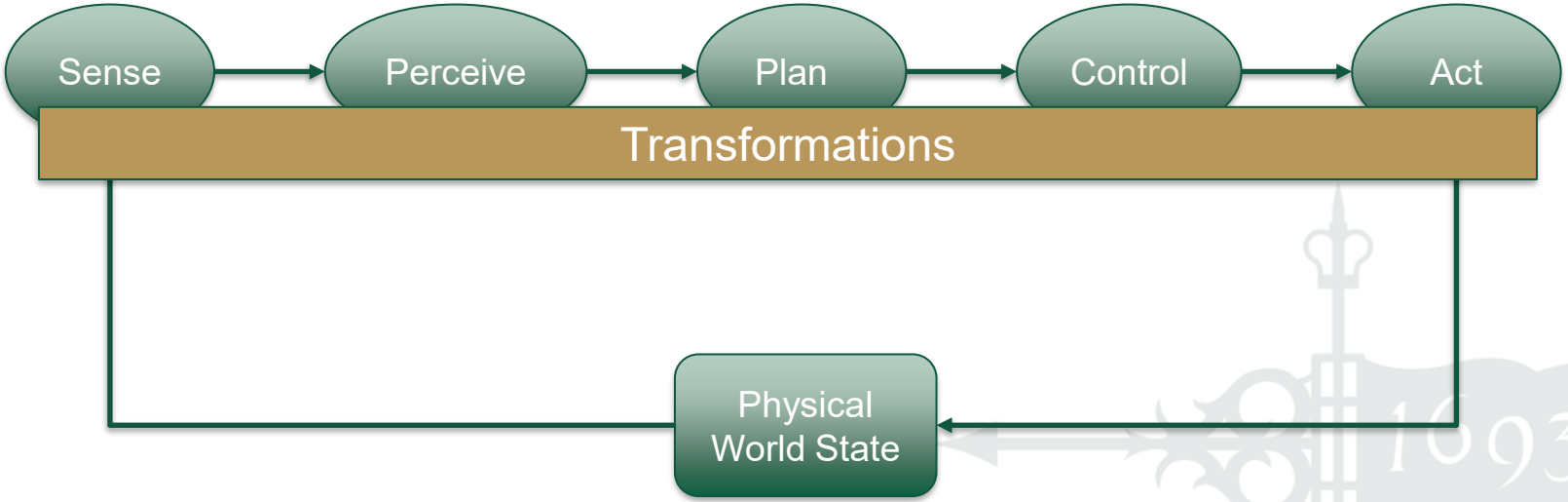
C5



Where are we now?

Another coordinate system?

Transformations



Coordinate Systems

- Method to “label” locations
- Associate numbers with points
- Need:
  - Origin
  - Basis Vectors (positive, unit length)

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  - Origin
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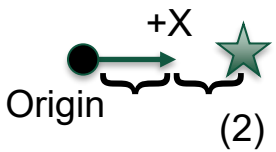
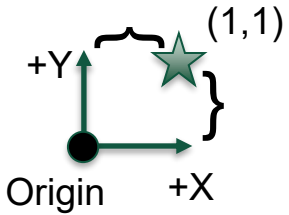
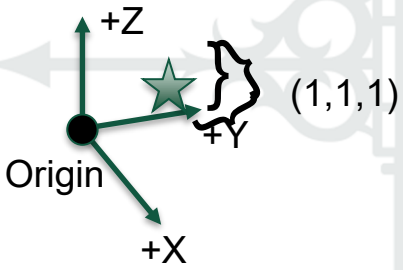
1D System



2D System

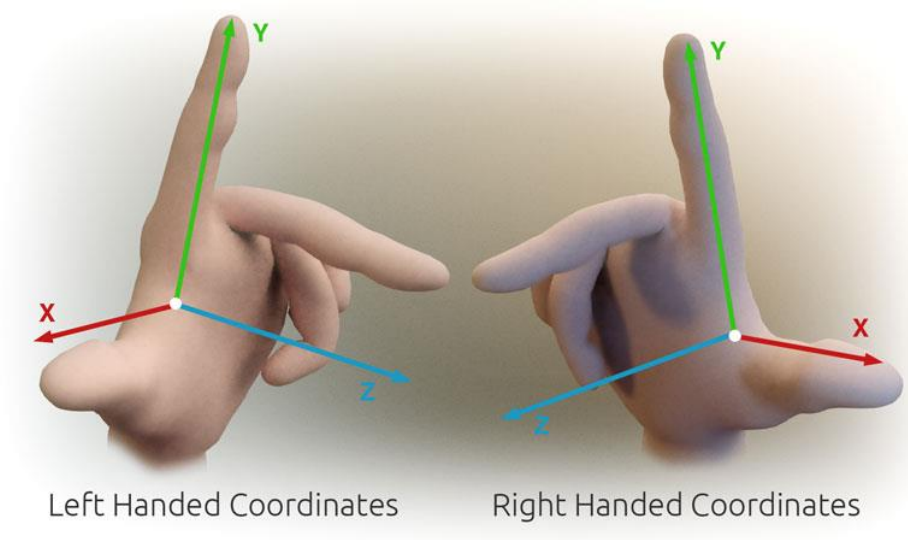


3D System



Coordinate System Conventions

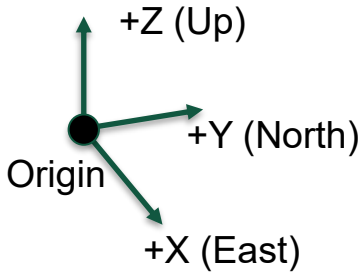
- Right-handed coordinate system
  - X – thumb
  - Y – pointer
  - Z – middle

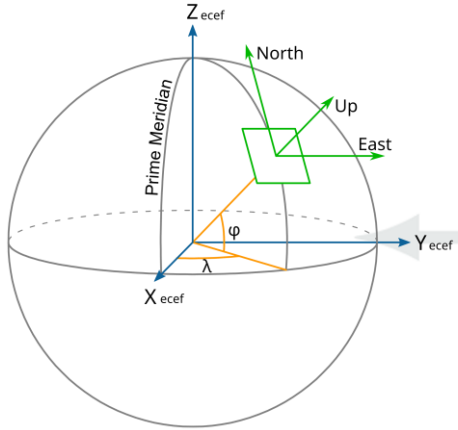


Multiple Conventions

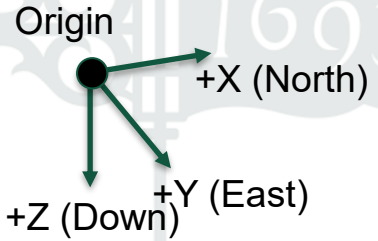
- 3D Reference Frames
- Multiple Conventions

ENU – East North Up



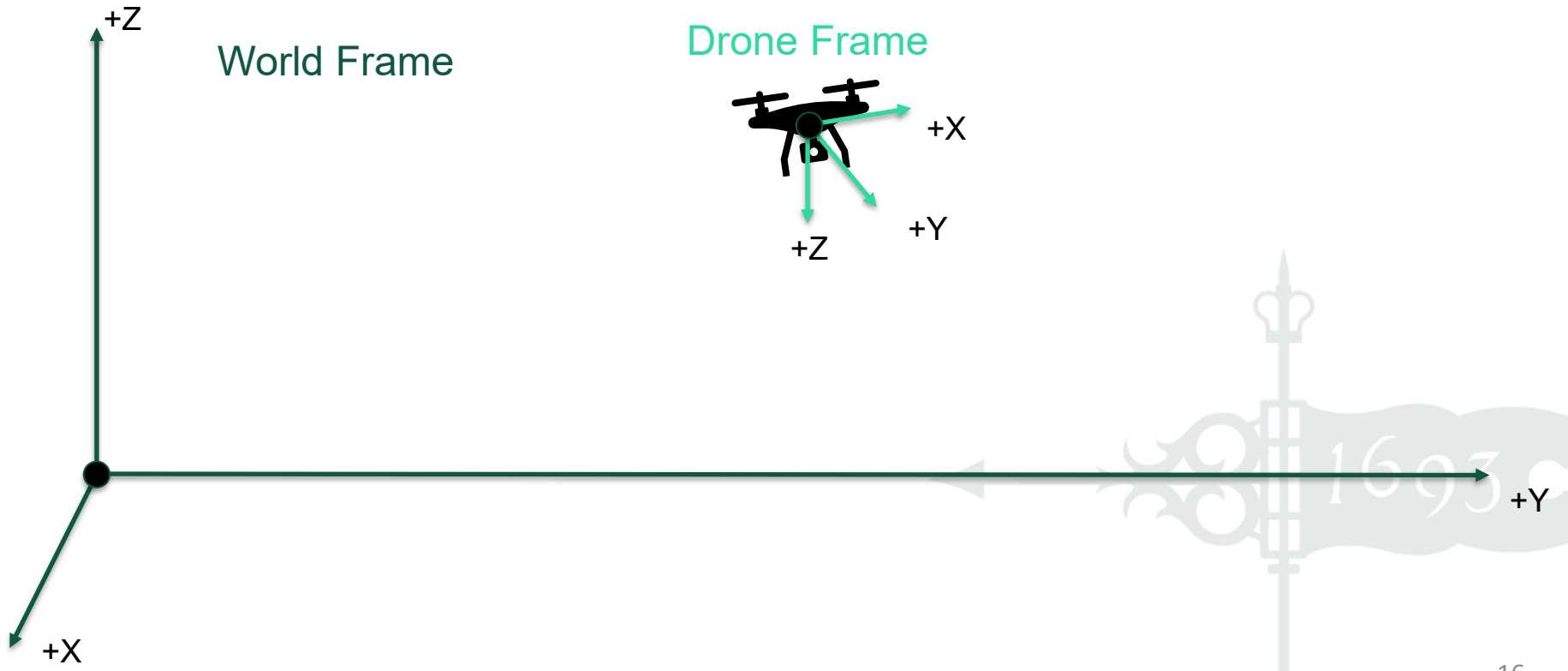


NED – North East Down

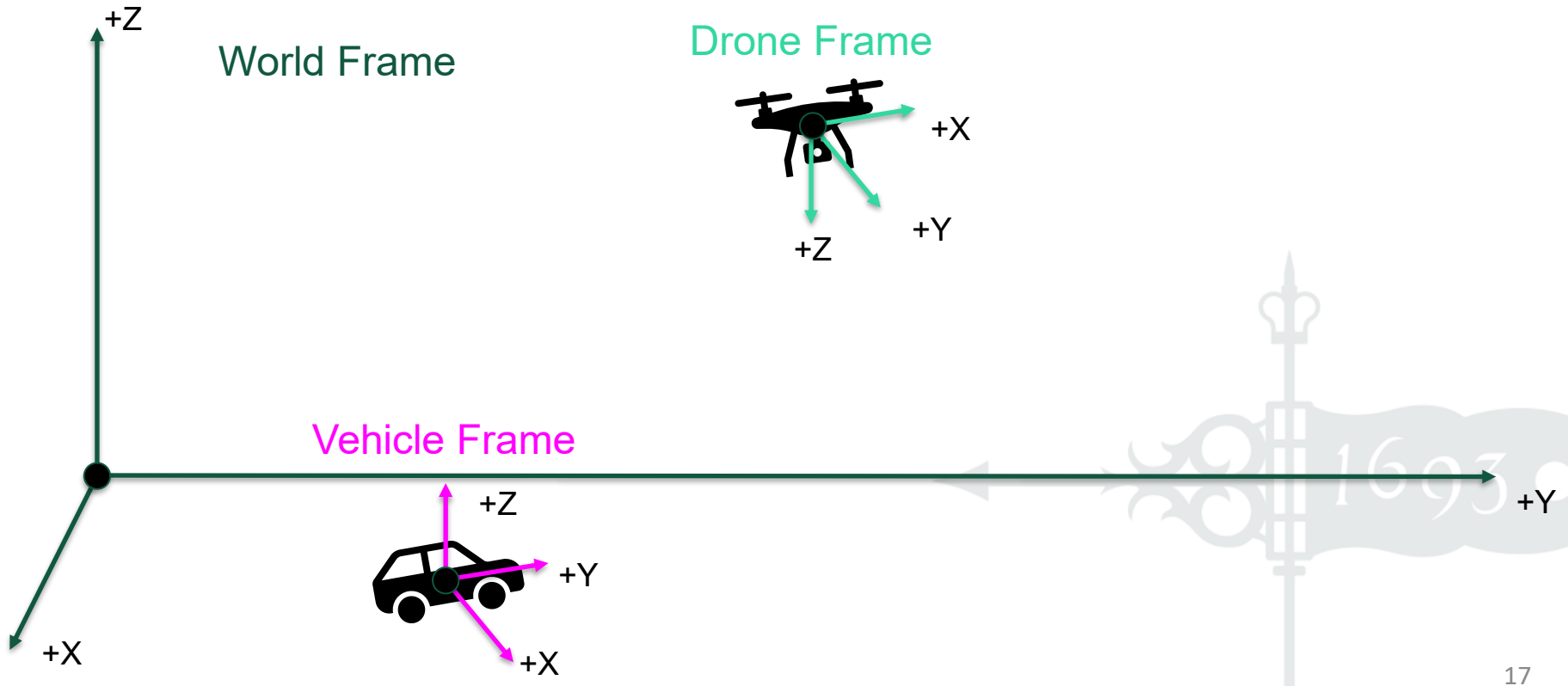


Multiple Coordinate Frames

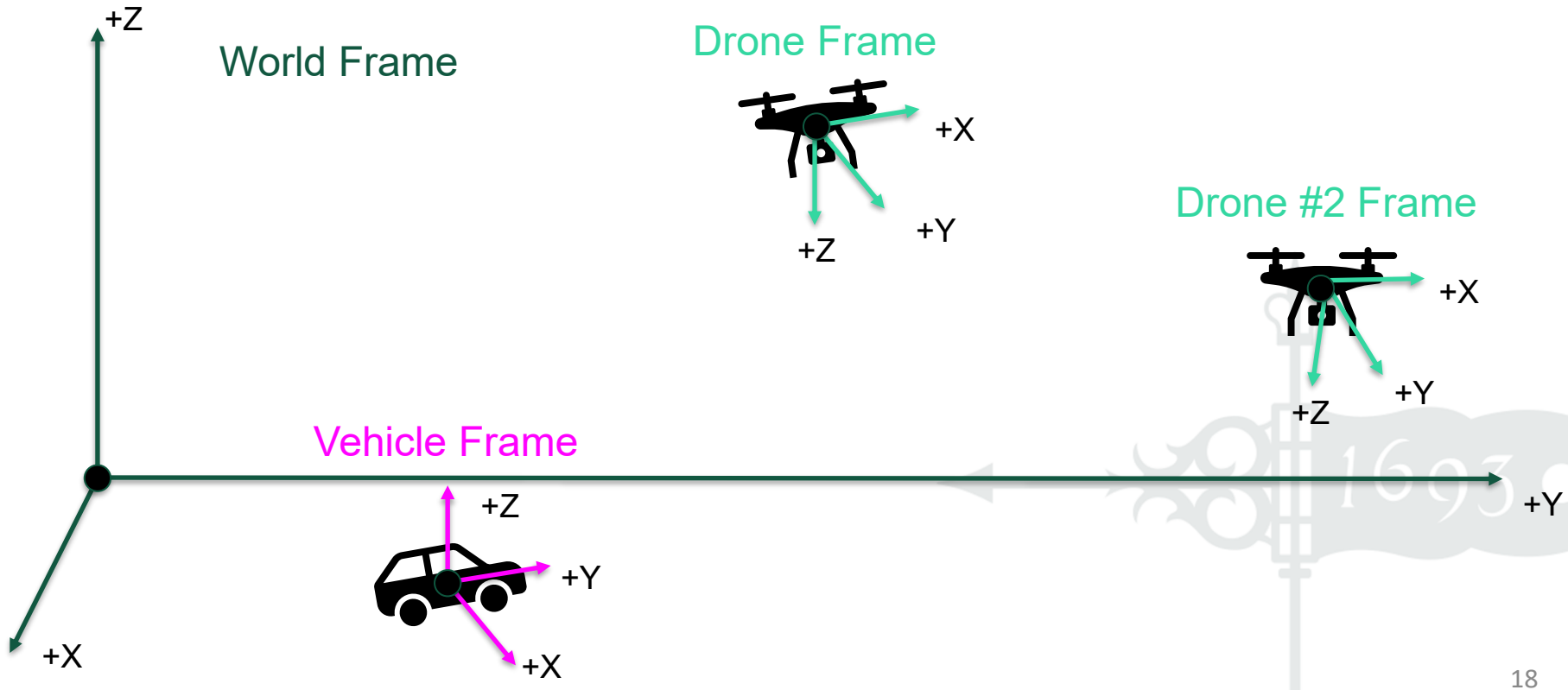
Multiple Coordinate Frames



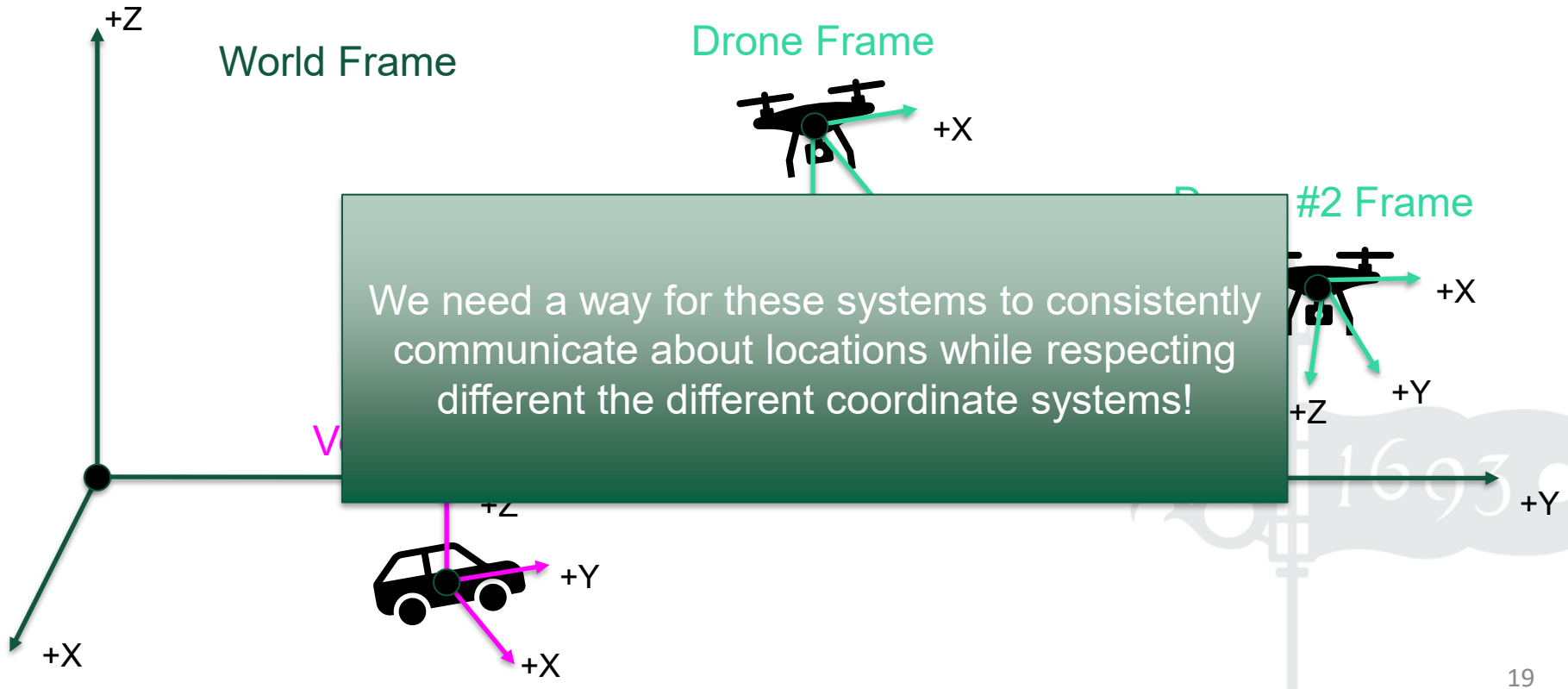
Multiple Coordinate Frames



Multiple Coordinate Frames



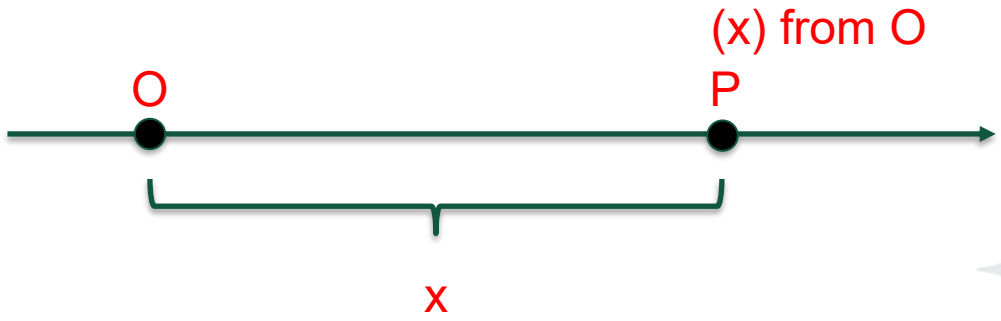
Multiple Coordinate Frames



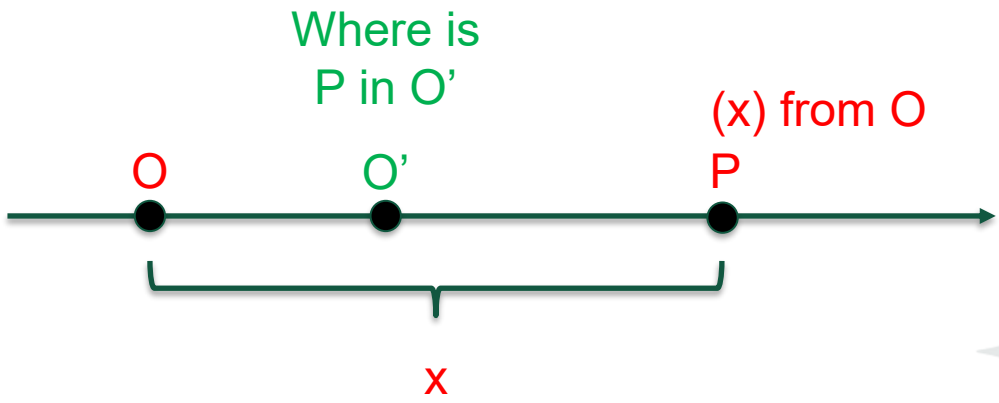
Transforms

- Function to “transform” from one coordinate system to another
  - Input: point/vector in frame A, target frame B
  - Output: point/vector in frame B
- Pseudocode:
  - Translate
  - Rotate

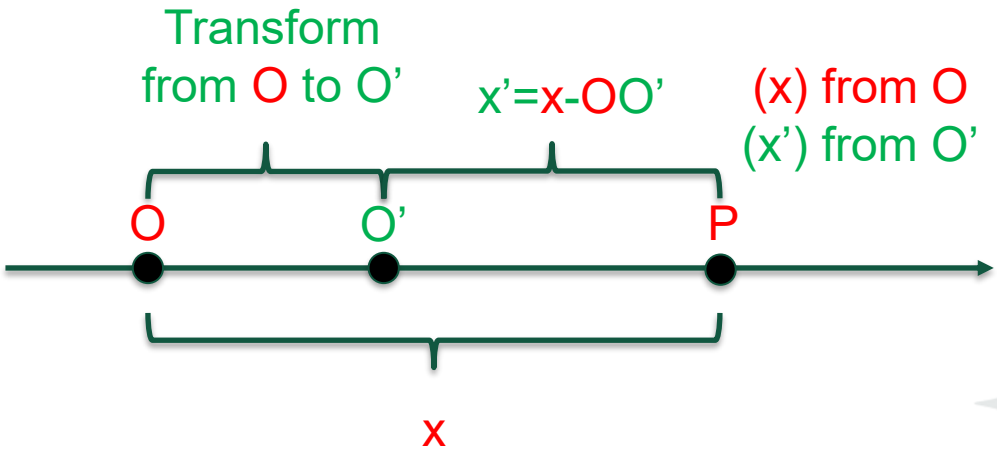
1D Transform



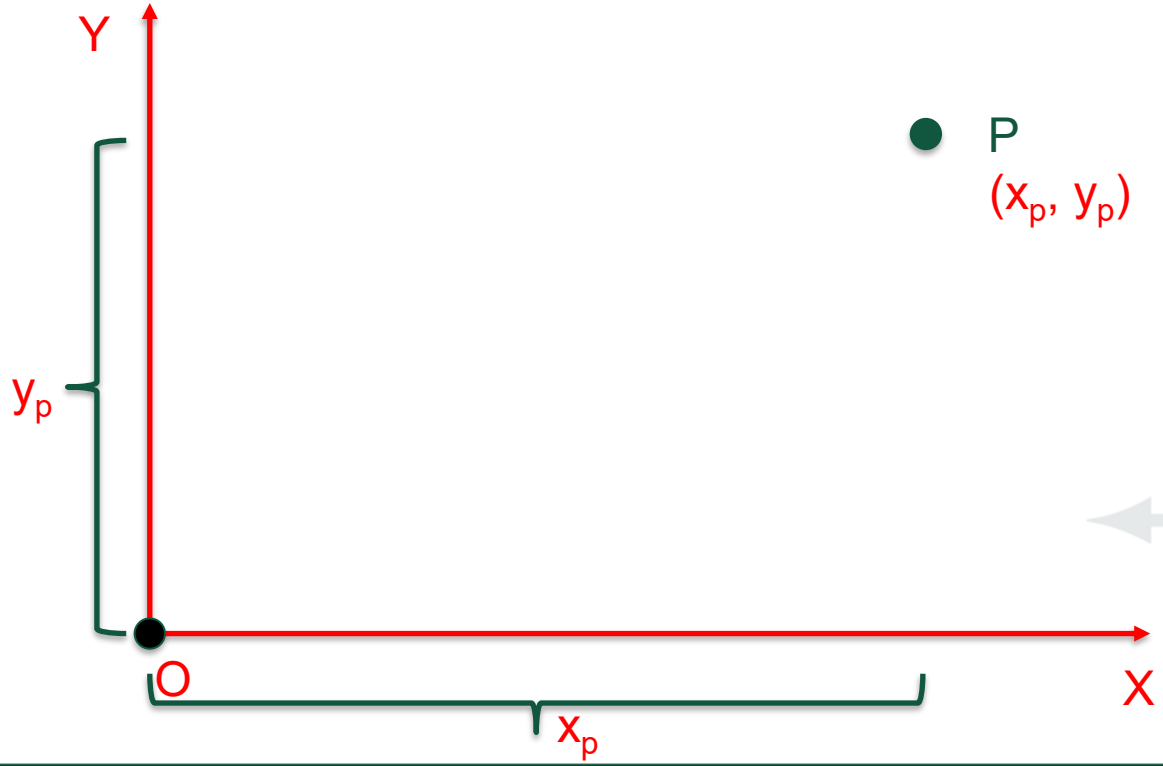
1D Transform



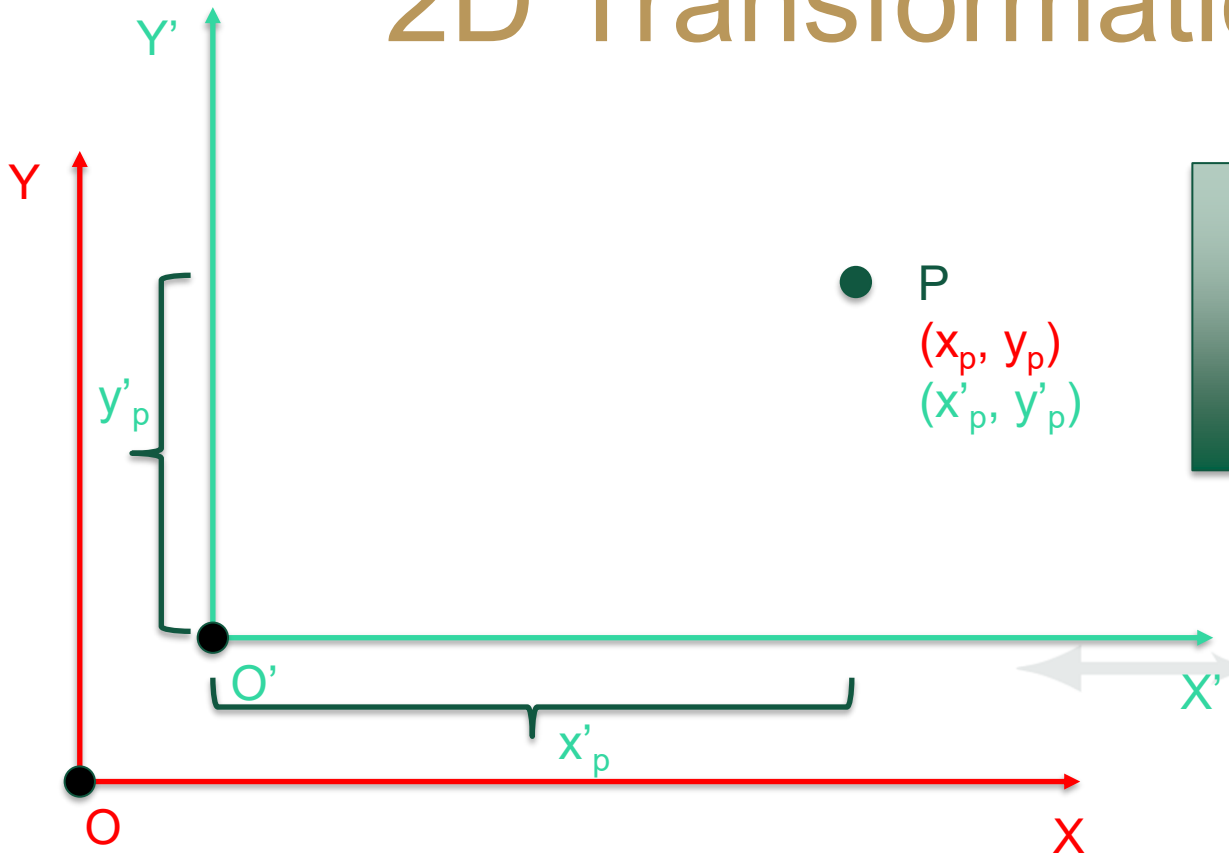
1D Transform



2D Transformation

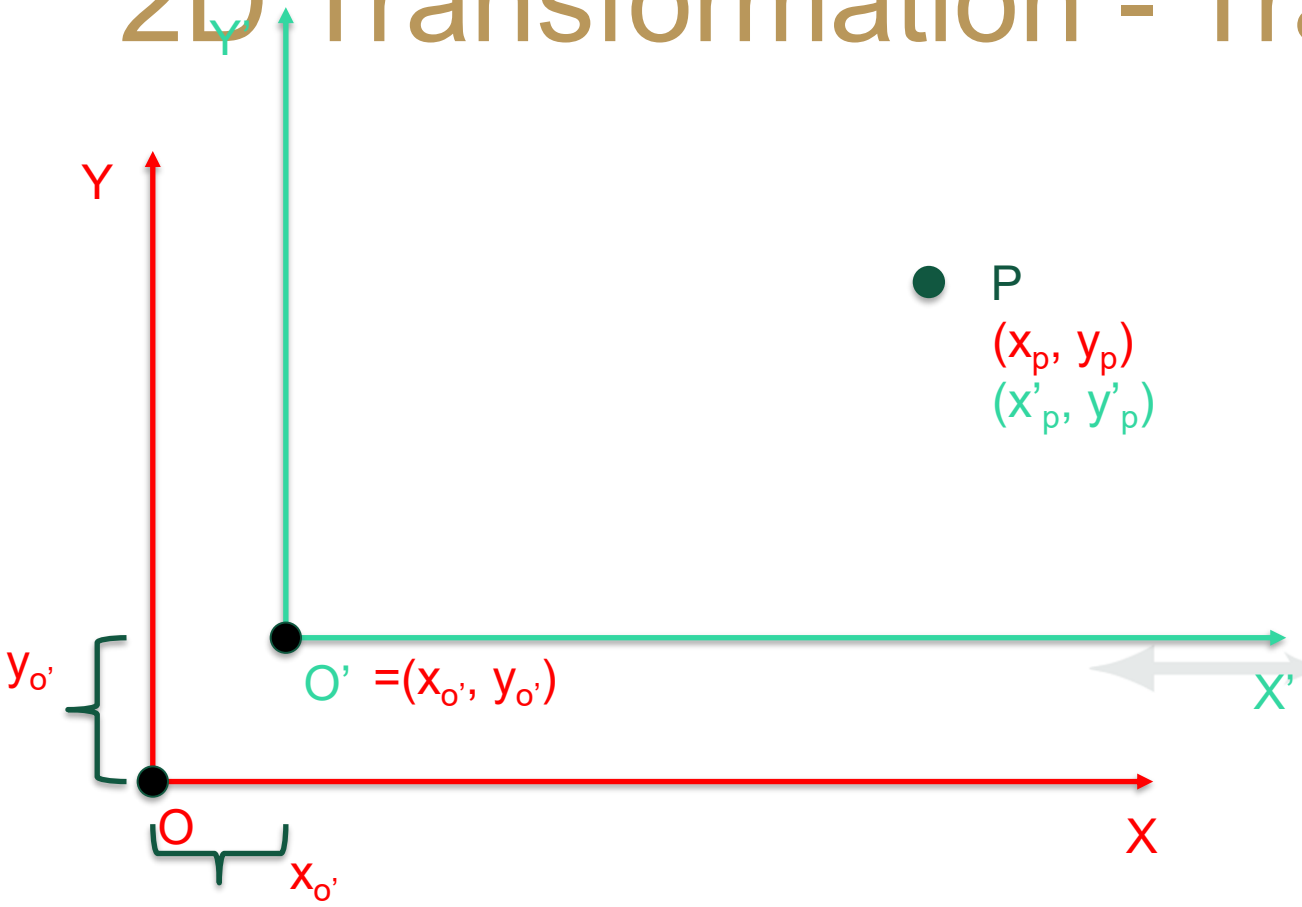


2D Transformation

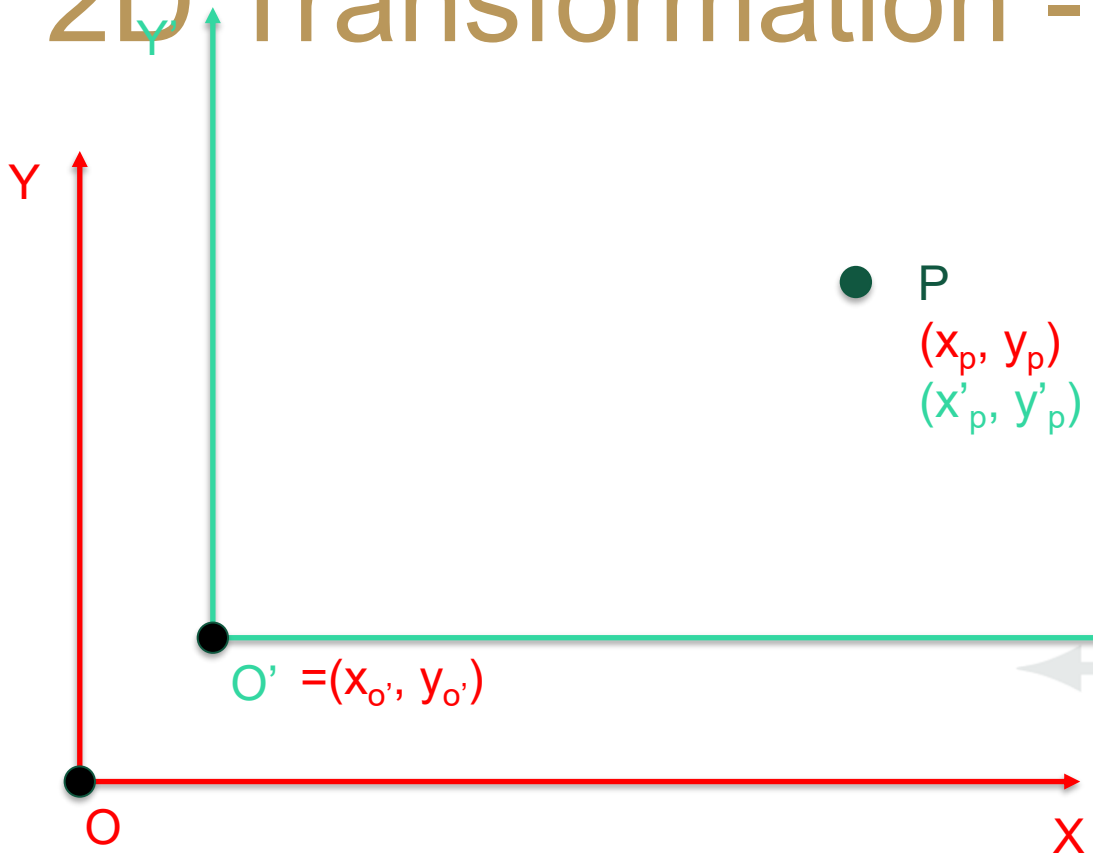


How do we relate these two coordinates so they can communicate?

2D Transformation - Translation



2D Transformation - Translation

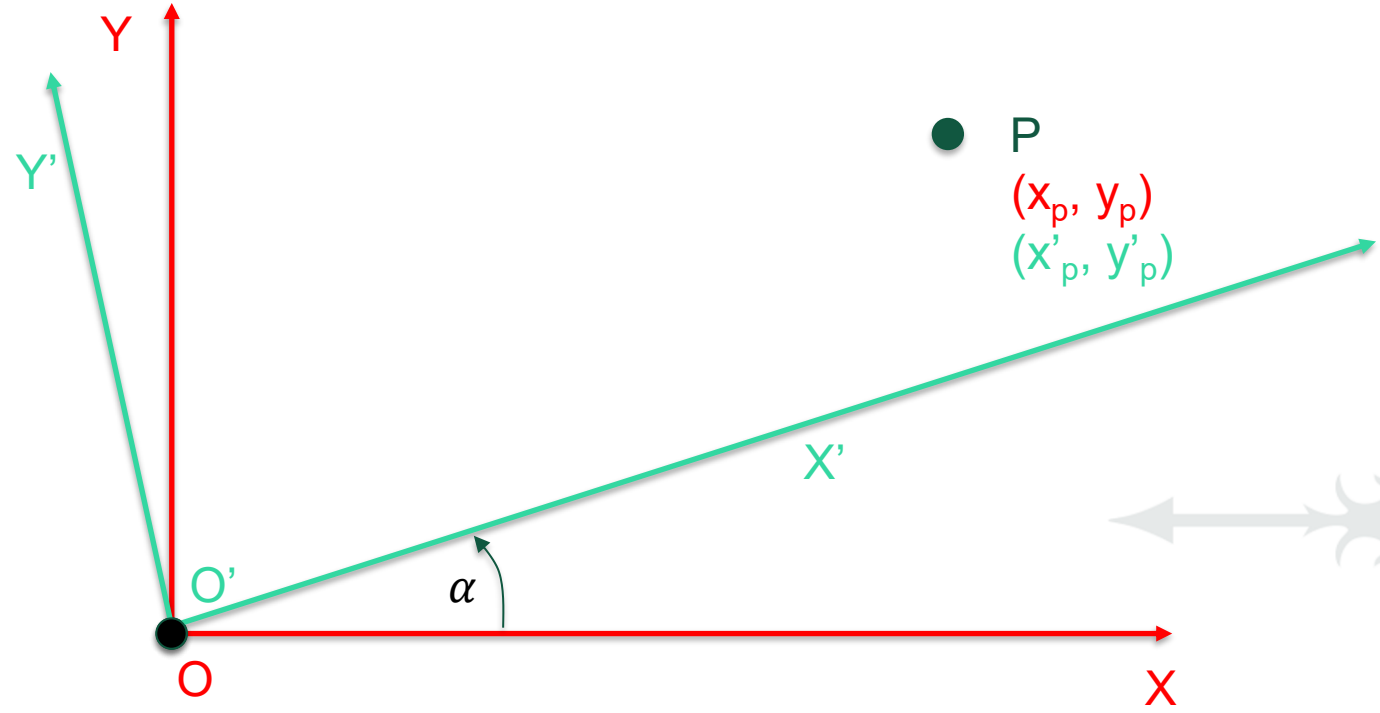


$$x'_p = x_p - x_{o'}$$
$$y'_p = y_p - y_{o'}$$

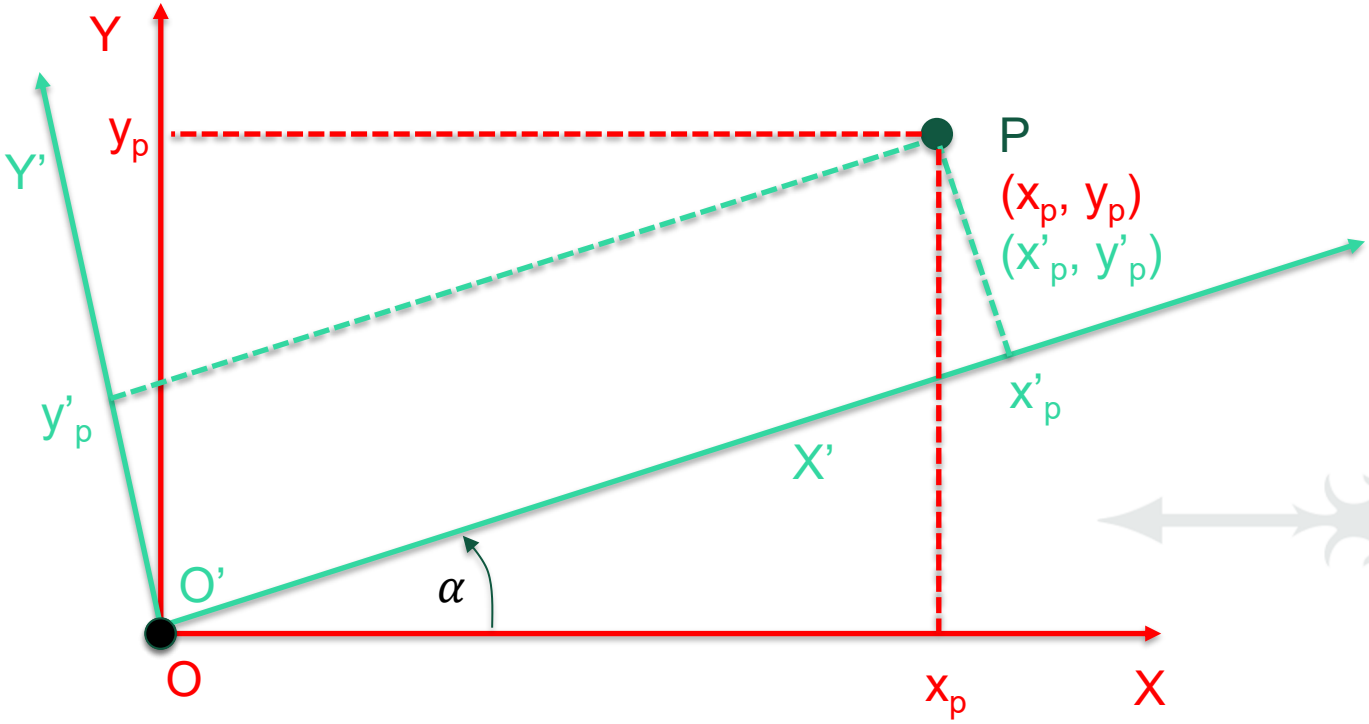
This is a system of equations!

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_o \\ y_o \end{bmatrix}$$

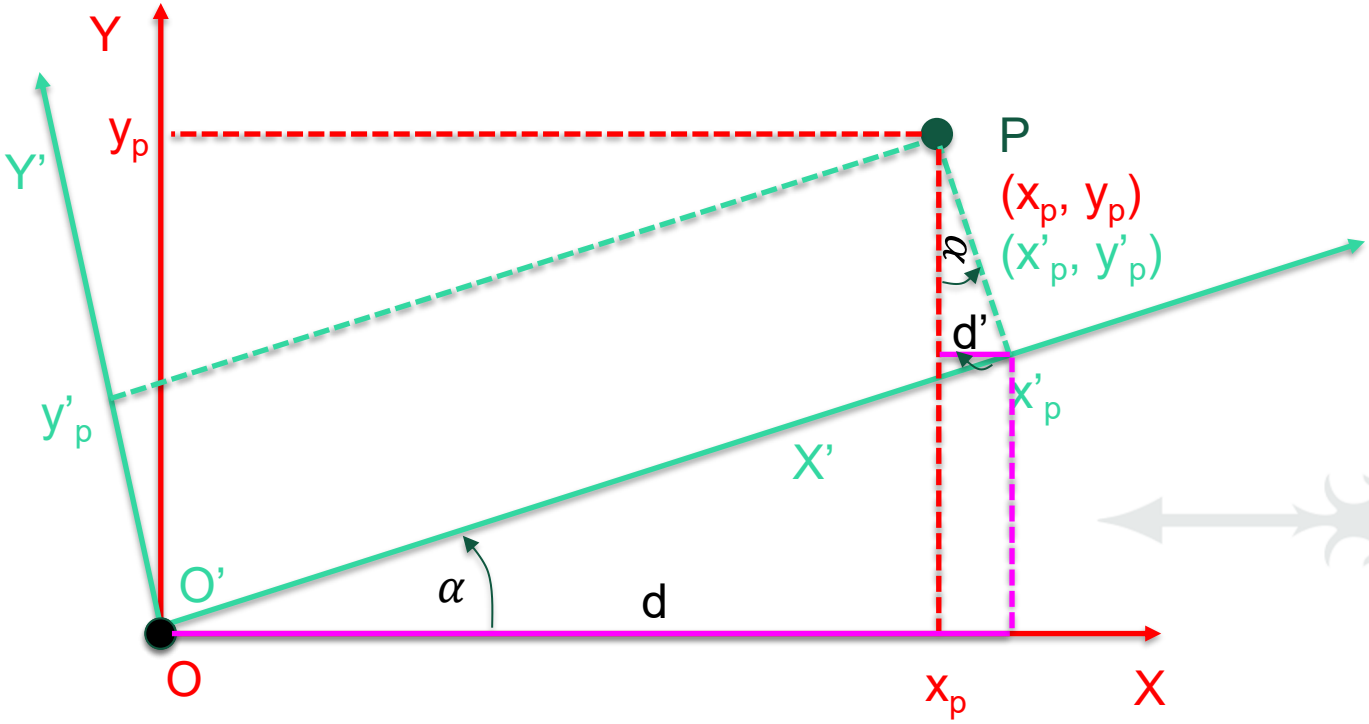
2D Transformation - Rotation



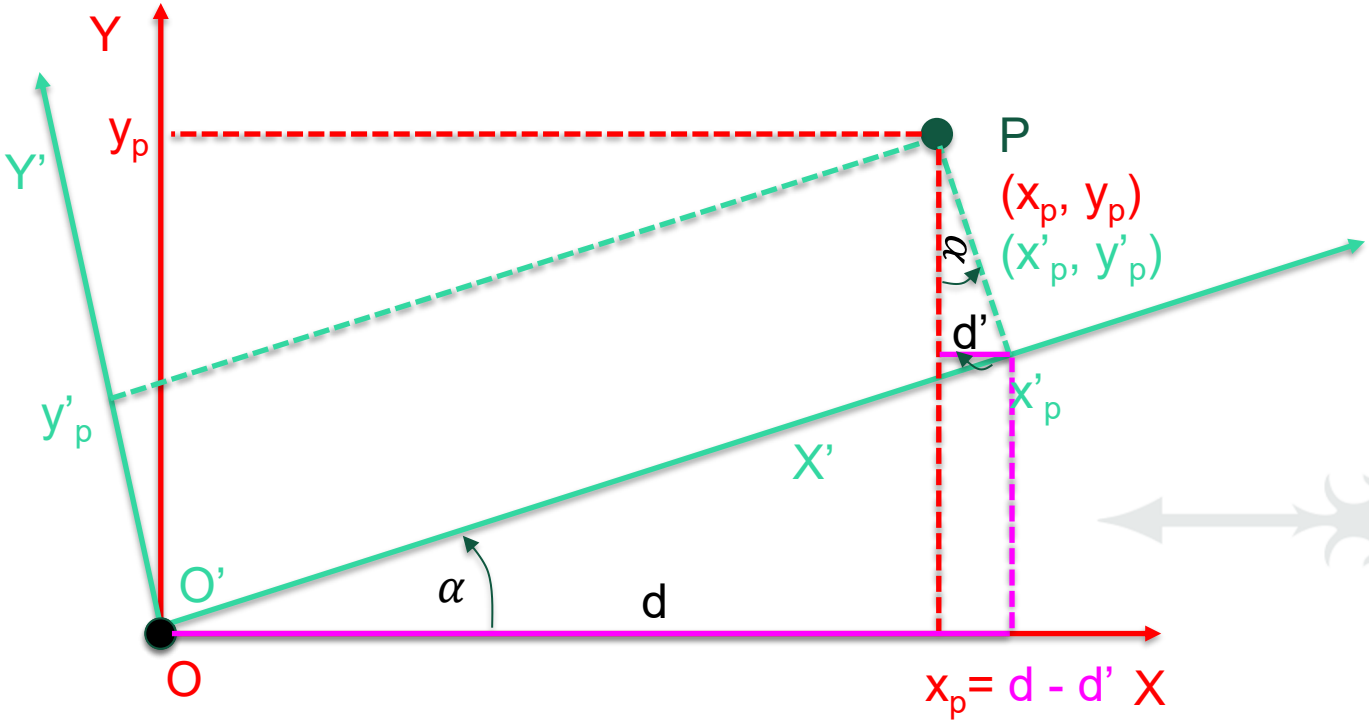
2D Transformation - Rotation



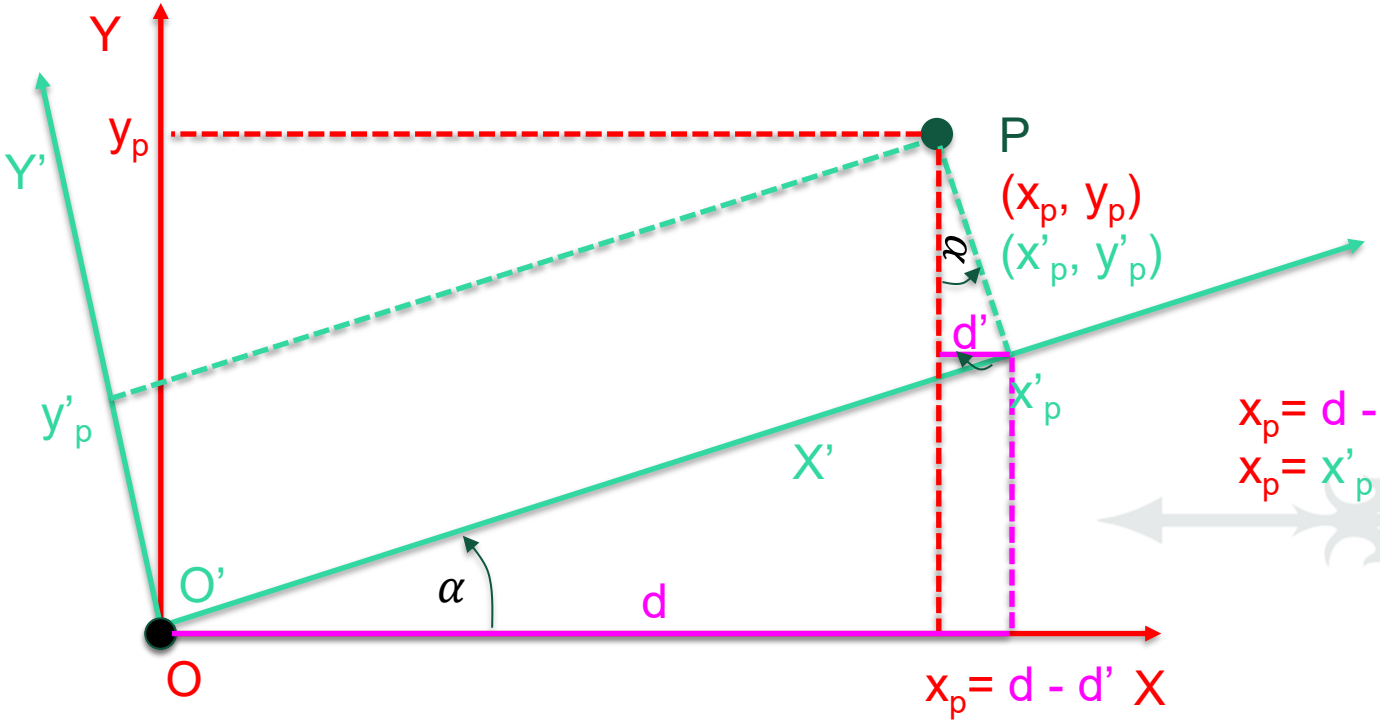
2D Transformation - Rotation



2D Transformation - Rotation



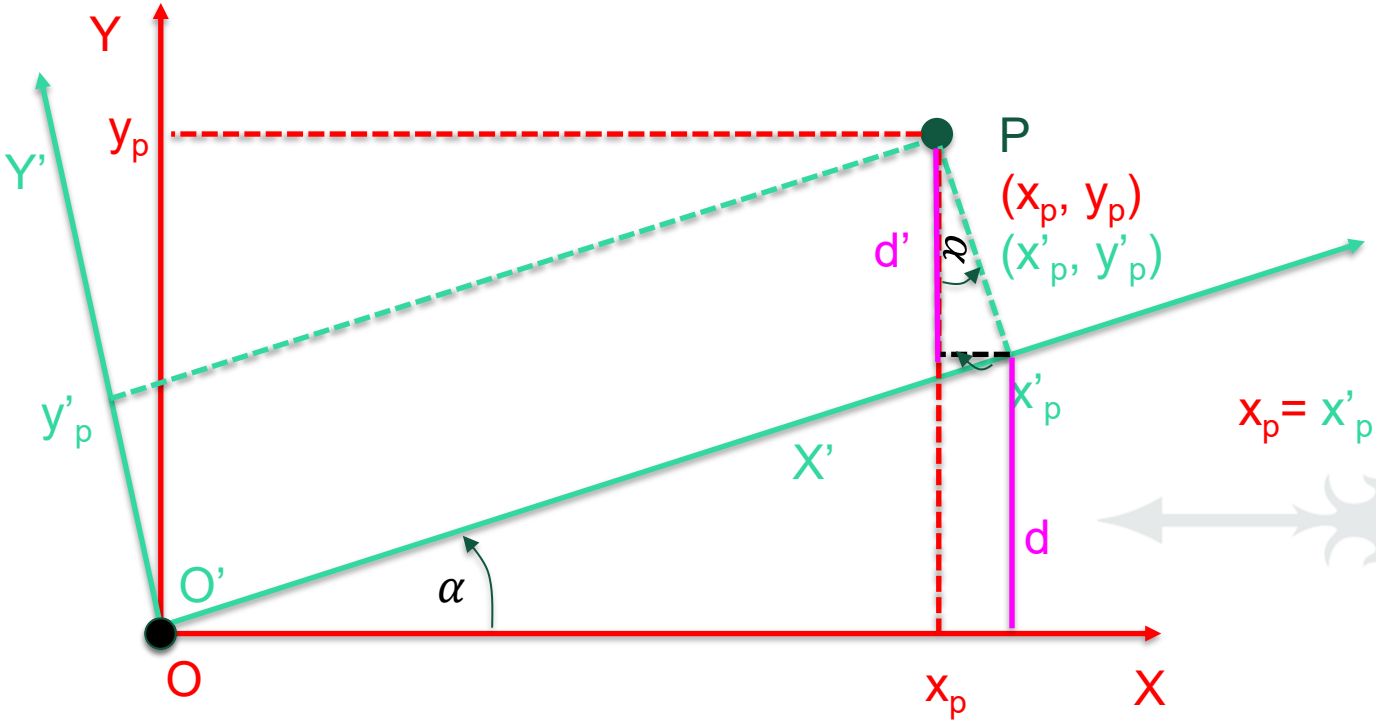
2D Transformation - Rotation



$x_p = d - d'$

$x_p = x'_p \cos(\alpha) - y'_p \sin(\alpha)$

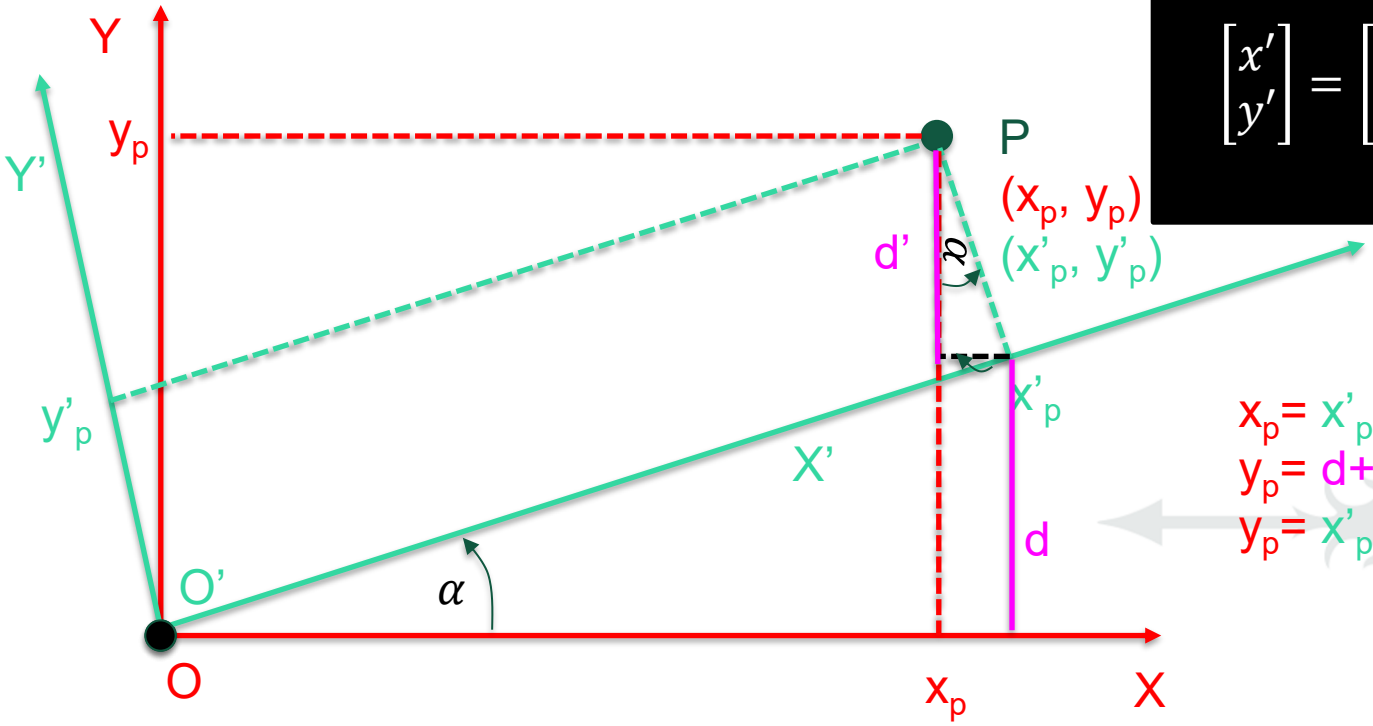
2D Transformation - Rotation



$$x_p = x'_p \cos(\alpha) - y'_p \sin(\alpha)$$

2D Transformation - Rotation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

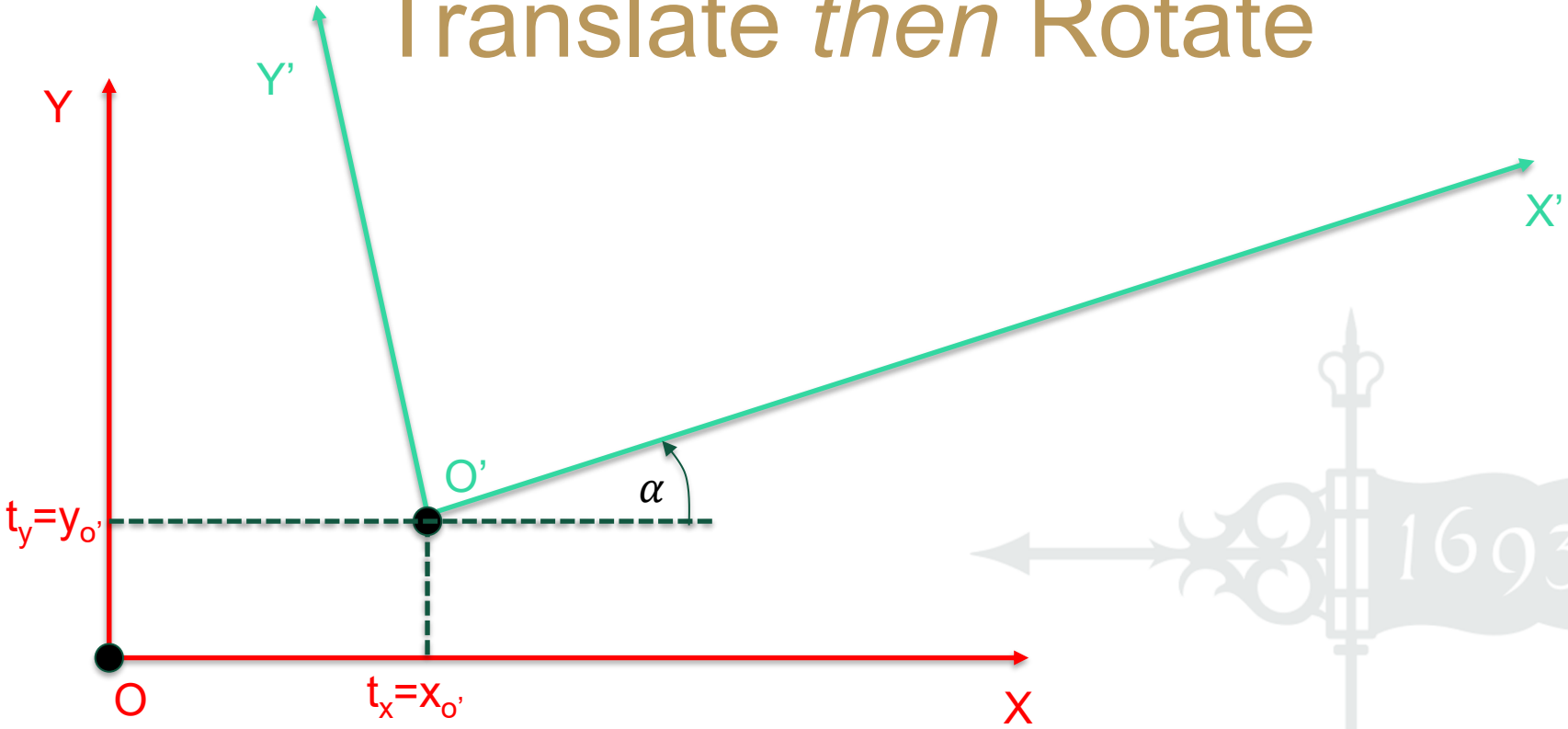


$x_p = x'_p \cos(\alpha) - y'_p \sin(\alpha)$

$y_p = d + d'$

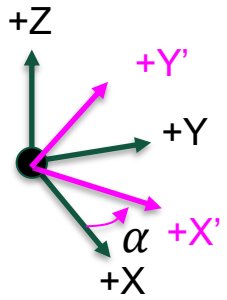
$y_p = x'_p \sin(\alpha) + y'_p \cos(\alpha)$

2D Transformation: Translate *then* Rotate



3D Transformations

- Rotation around Z



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

3D Transformations

- Rotation around X, Y, Z all different
  - Composition of rotations
  - Multiplication of matrices is **not commutative**
  - Must agree on order

Rotation about X

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Rotation about Y

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

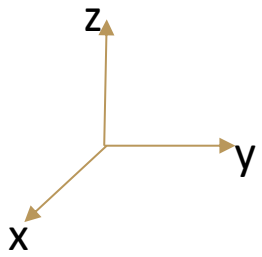
Rotation about Z

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

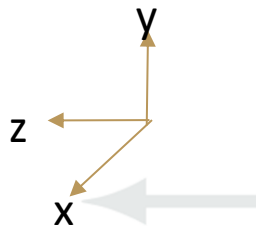
3D Transformations

- Rotation around X, Y, Z all different
  - Composition of rotations
  - Multiplication of matrices is **not commutative**
  - Must agree on order

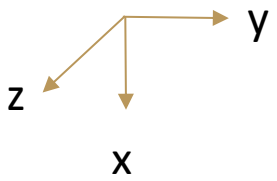
90 degrees
counter-clockwise



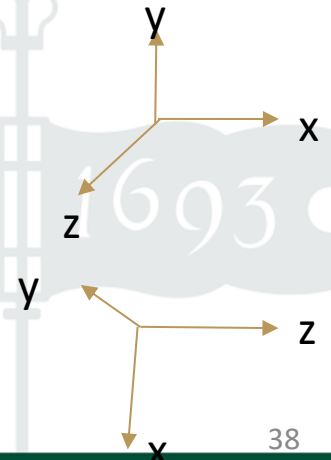
Rx



Ry



Ry



Rx

3D Transformations

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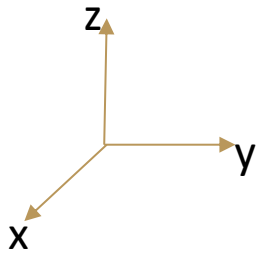
90 degrees counter-clockwise



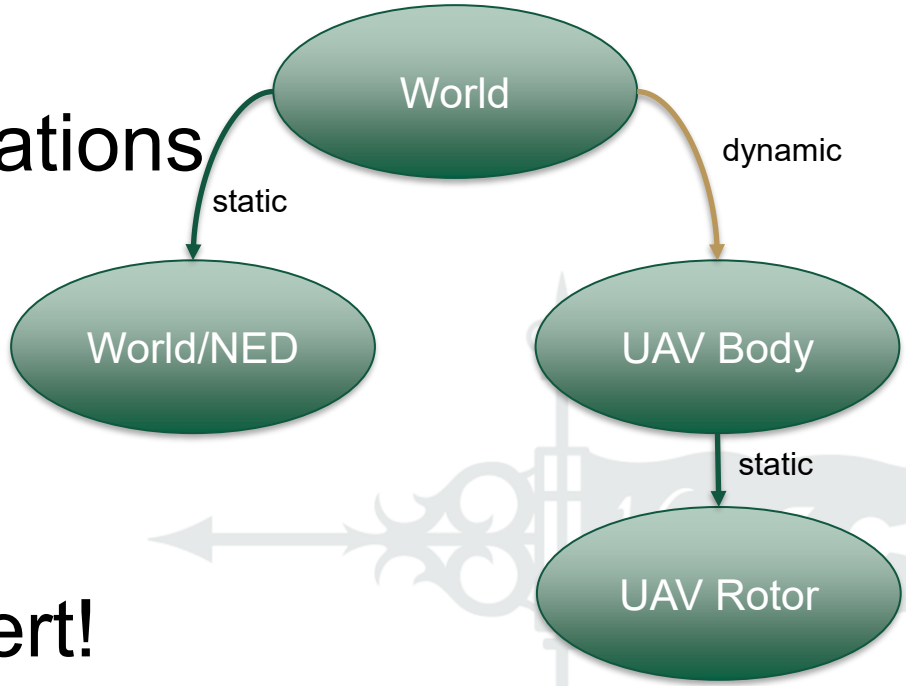
Rx

Ry

ROS has transformation libraries that automatically handle the math from one coordinate frame to another!

Frames in ROS

- tf2 api
- Define frames and relations
  - Tree!
- Transforms between
  - Static
  - Dynamic
- Helper library to convert!



Transformations

- Physical data without a coordinate frame is **useless!**

- The frame is part of the data type!